Ivan Bratko

PROLOG PROGRAMMING FOR ARTIFICIAL INTELLIGENCE, 4TH EDN.
ADDITIONAL EXERCISES IN BAYESIAN NETWORKS

Note: These exercises are supplementary to those in the book: I. Bratko, Prolog Programming for Artificial Intelligence, 4th edn., Pearson Education 2011.

Problem

Let there be variables X and Y in a dynamic system. The behaviour of X in time is:

\[ X = \sin(t) \]

(a) What is the corresponding qualitative behaviour in time of X? Give the first 7 qualitative states, starting with \( t = 0 \).

(b) Let there be the qualitative constraint \( Y = M'(X) \), and the initial value \( Y(0) \geq 0 \). Give all the possible qualitative behaviours of both X and Y in time, starting with \( t=0 \) (the first 7 qualitative states).

Answers

(a) zero/inc, zero..inf/inc, zero..inf/std, zero..inf/dec, zero/dec, minf..zero/dec, minf..zero/std

(b)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>zero/inc</td>
<td>zero..inf/inc</td>
</tr>
<tr>
<td>zero..inf/inc</td>
<td>zero..inf/inc</td>
</tr>
<tr>
<td>zero..inf/std</td>
<td>zero..inf/std</td>
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<tr>
<td>zero..inf/dec</td>
<td>zero..inf/dec</td>
</tr>
<tr>
<td>zero/dec</td>
<td>zero..inf/dec</td>
</tr>
<tr>
<td>minf..zero/dec</td>
<td>zero..inf/dec</td>
</tr>
<tr>
<td>minf..zero/std</td>
<td>zero..inf/std or zero/std</td>
</tr>
</tbody>
</table>

Also possible:
7. minf..zero/dec zero/dec
Problem
Consider a QDE-type (Qualitative Differential Equations) qualitative model with three variables $x$, $y$ and $z$, and the following constraints:

\[
M_0^{\prime} (x, y) \quad \text{(monotonically decreasing functions; if } x=0 \text{ then } y=0) \\
\text{plus}(y, z, x) \quad \text{(} y + z = x \text{)} \\
\text{deriv}(y, z) \quad \text{(} dy/dt \text{ is “qualitatively equal” } z \text{)}
\]

The landmarks for these variables, ordered from smallest to largest, are:

- $x$: minf, $0$, $x_0$, $x_1$, $x_2$, inf
- $y$: minf, $y_0$, 0, inf
- $z$: minf, $0$, $z_0$, $z_1$, $z_2$, inf

At time $t_0$ the qualitative values of $x$, $y$ and $z$ are equal $x_1$, $y_0$ and $z_1$, respectively.

(a) What are all the possible qualitative states (i.e. values and directions of change) of these three variables at the initial time $t_0$.
(b) What are all the possible qualitative states of the three variables in the time interval $t_0..t_1$?
(c) What are all the possible qualitative states of the three variables at time $t_1$?
(d) Can the system reach, from the given initial state at time $t_0$, a steady state (that is a state where all the variables are constant – not changing in time – and from that point stay the same for ever. If yes, what is this steady state?

Answers
In answering the questions, the representation below helps. The horizontal arrows indicate the directions of change at time $t_0$.

\[
\begin{array}{c}
M_0^{\prime} \quad \xymatrix{ x \ar[dr] & 0 & x_0 & x_1 & x_2 \ar[dl] \ar[dr] \\
y \ar[dr] & y_0 & 0 \ar[dl] \ar[dr] \\
z \ar[dr] & 0 & z_0 & z_1 & z_2 \ar[dl] \\
} \\
d/dt \\
\end{array}
\]

In addition to the graphically indicated constraints, it also holds: $x = y + z$.

(a) The qualitative state at time $t_0$ is as indicated in the diagram.
(b) It is easy to see from the diagram that all the qualitative values are corresponding intervals (e.g. $x = x_0..x_1$) and the directions of change are the same as at time $t_0$, as indicated by the horizontal arrows.
(c) There are three alternatives possible: (1) $x$ reaches $x_0$, or (2) $z$ reaches $z_0$, or (3) both $x$ and $z$ reach these landmarks simultaneously. The directions of change remain the same. $y$ cannot become std because $z > 0$. In all the three cases for the values of $x$ and $z$, $y = y_0..0$. $y$ cannot be $0$ because $x > 0$. 

2
(d) The possible stationary state is \( x = y = z = 0 \), all std.

Problem
Suppose we have a qualitative model with variables \( X \), \( Y \) and \( Z \). The landmarks for all three variables are:

\[
\text{minf, 0, inf}
\]

where ‘inf’ stands for infinity, ‘minf’ stands for minus infinity. There are the following constraints in the model:

\[
\begin{align*}
Y &= M_0^+(X) \\
Z &= M_0^-(Y) \\
\text{deriv}(X,Z) &= \text{dX/dt is “qualitatively equal” Z}
\end{align*}
\]

At the initial time \( t_0 \), the qualitative value of \( X \) is 0..inf.

(a) What are the qualitative values of \( Y \) and \( Z \) at \( t_0 \)?
(b) What is the direction of change (inc, std, or dec) of \( X \) at \( t_0 \)? Explain why.
(c) What are the qualitative states (that is qualitative values and directions of change) of the three variables in time interval \( t_0..t_1 \)?
(d) What are all possible qualitative states of the three variables at time point \( t_1 \)?

Problem
Let there be three variables \( x \), \( y \) and \( z \) in a QDE-type qualitative model. Landmark values for these variables are:

\[
\begin{align*}
x &: \text{minf, 0, x0, inf} \\
y &: \text{minf, 0, y0, inf} \\
z &: \text{minf, 0, inf}
\end{align*}
\]

“minf” means minus infinity, “inf” means infinity.

There are the following constraints in the model:

\[
\begin{align*}
y &= M_0(x) & (y \text{ is monotonically decreasing function of } x, \text{ with } y = 0 \text{ when } x = 0) \\
\text{plus}(x,z,y) &= (x + z = y) \\
\text{deriv}(x,z) &= (\text{dx/dt is qualitatively equal } z)
\end{align*}
\]
At time $t_0$, the qualitative value of $x$ is: $x(t_0) = x_0/\text{dec}$

(a) What are all possible qualitative values $y(t_0)$ and $z(t_0)$ that satisfy the above qualitative constraints?
(b) What are all possible qualitative values of $x$, $y$ and $z$ in the interval $t_0..t_1$ that immediately follows time $t_0$.
(c) Is it possible that this system gets into a steady state at time point $t_1$ (all variables are steady)? If yes, what are the qualitative values of $x$, $y$ and $z$ in this steady state?

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Problem
Let there be three variables $x$, $y$ and $z$ in a QDE-type qualitative model. Landmark values for these variables are:

- $x$: $\text{minf}$, 0, $x_0$, $\text{inf}$
- $y$: $\text{minf}$, $y_0$, 0, $\text{inf}$
- $z$: $\text{minf}$, 0, $\text{inf}$

“$\text{minf}$” means minus infinity, “$\text{inf}$” means infinity.

There are the following constraints in the model:

- $y = M_0(x)$ \hspace{1cm} \% $y$ is monotonically decreasing function of $x$, with $y=0$ when $x=0$
- $\text{plus}(x,z,y)$ \hspace{1cm} \% $x + z = y$
- $\text{deriv}(x,z)$ \hspace{1cm} \% $\text{dx/dt} = z$

At time $t_0$ the qualitative value of $x$ is: $x(t_0) = x_0..\text{inf}/\text{dec}$.
It is also given that the value of $y(t_0)$ is less than $y_0$, but the direction of change (inc, dec, std) of $y$ at $t_0$ is not given.

(a) What are all possible qualitative values $y(t_0)$ and $z(t_0)$ that satisfy the above qualitative constraints?
(b) What are all possible qualitative values of $x$, $y$ and $z$ in the interval $t_0..t_1$ that immediately follows time $t_0$?
(c) What are all possible qualitative values of $x$, $y$ and $z$ at time point $t_1$?

______________________________________________________________________________

Problem
Let there be four variables, $X$, $Y$, $Z$ and $D$, in a QDE-type qualitative model. The landmarks for all these variables are: $\text{minf}$, 0, $\text{inf}$ (minus infinity, zero and infinity). The landmarks for $D$ are: $\text{minf}$, 0, $d_0$, $\text{inf}$. The model consists of the following constraints:
\[ \text{deriv}( Z, D) \quad (D \text{ is qualitatively equal } dZ/dt) \]
\[ \text{add}( X, Y, Z) \quad (X+Y = Z) \]
\[ Y = M'(X) \quad (Y \text{ is monotonically decreasing function of } X) \]
\[ D = \text{const.} \quad (D \text{ does not change}) \]

At the initial time \( t_0 \), the values of the variables are \( X = Y = Z = \text{zero} \), \( D = d_0 \).

(a) What are all possible qualitative states (that is qualitative values and directions of change) of the four variables at time \( t_0 \) so that they satisfy the given constraints?

(b) What are all possible qualitative states of the four variables in the time interval between \( t_0 \) and \( t_1 \), so that they satisfy the given constraints?

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**Problem**

There are three variables \( x, y \) and \( z \) in a qualitative model which includes the following constraints:

\[
M'(x, y) \\
\text{plus}(y, z, x) \\
\text{deriv}(y, z)
\]

The landmark values for the variables are:
\[
\begin{align*}
x: & \text{ minf, 0, } x_0, \text{ inf} \\
y: & \text{ minf, 0, } y_0, \text{ inf} \\
z: & \text{ minf, 0, inf}
\end{align*}
\]

In the initial state of the system at time \( t_0 \), the qualitative values of \( x \) and \( y \) are \( x_0 \) and \( y_0 \).

(a) Determine all possible qualitative states (that is qualitative values and directions of change) of the three variables at time \( t = t_0 \).

(b) What are all possible qualitative states of the three variables in the time interval \( t_0..t_1 \)?

(c) What is the smallest qualitative value that the variable \( y \) can reach from the initial state?

**Answers**

The following answers were obtained with the Prolog qualitative simulation program in the book:

(a)
\[
\begin{align*}
S_0 &= [ x:x0/\text{dec}, y:y0/\text{inc}, z:\text{zero..inf}/\text{dec} ] \\
S_0 &= [ x:x0/\text{std}, y:y0/\text{std}, z:\text{zero/}\text{std} ] \\
S_0 &= [ x:x0/\text{inc}, y:y0/\text{dec}, z:\text{minf..zero/}\text{inc} ] \\
\end{align*}
\]
no

(b)
S1 = [ x:zero..x0/dec, y:y0..inf/inc, z:zero..inf/dec] ? ;
S1 = [ x:x0..inf/inc, y:zero..y0/dec, z:minf..zero/inc] ? ;
no

Another possibility is for the variables to stay unchanged for ever:

S1 = [ x:x0/std, y:y0/std, z:zero/std]

The qualitative simulator in the book does not produce this state because there is no change from the previous steady state (at t₀).

(c) y = zero..y0 (y can never be less or equal 0)